Name............................... **Roll No.........................**

Department of Mathematics
Guru Nanak Dev University, Amritsar-143005
Quiz Test in Mathematics II: B.Sc.(Hons.)Physics Semester II

April, 2011 **Max. Marks 10**
Course Code#MTL132 **Time Allowed: 60 Minutes**

Note: Read the instructions carefully:
★ Attempt all 20 questions by ticking ✓ *only* one of the four choices (a), (b), (c), and (d) for each question below.
★ Response to any question marked for more than one choice will not be counted for any score.

1. Let $L(y) = 0$ be a linear ODE of order 10 with analytic variable coefficients. Then it has
(a) 10 different solutions (b) no solution
(c) 10 linearly dependent solutions (d) 10 linearly independent solutions

2. Let $\{\varphi_1, \ \ldots, \varphi_n\}, \ n \geq 2$ be a basis for the solution space of $L(y) := 0$. Then any solution $\varphi(x)$ satisfying $L(\varphi) = 0$ is given by
(a) $\varphi(x) = \sum_{i=1}^{\infty} c_i \varphi_i(x)$ (b) $\varphi(x) = \sum_{i=1}^{n-1} c_i \varphi_i(x)$ (c) $\varphi(x) = \sum_{i=1}^{n} c_i \varphi_i(x)$ (d) none

3. Two linearly independent solutions of $2xy'' - 2y' + \dfrac{2}{1+x} y = 0, \ x > -1$ are
(a) $\varphi_1 = 1 + x, \ \varphi_2 = 1 + (1-x)\log(1-x)$ (b) $\varphi_1 = 1 + x, \ \varphi_2 = \dfrac{1}{1-x}$
(c) $\varphi_1 = 1 + x, \ \varphi_2 = 1 + (1+x)\log(1+x)$ (d) $\varphi_1 = 1 + x, \ \varphi_2 = \dfrac{1}{1+x}$.

4. Let φ_1 and φ_2 be two linearly independent solutions of $y'' - 2(x-1)y' + x^2 y = 0, \ x > 0$ such that the Wronskian $W(\varphi_1, \ \varphi_2)(1) = \xi$ for some $\xi \in \mathbb{R}$. Then
(a) $W(\varphi_1, \ \varphi_2)(2) = \xi$ (b) $W(\varphi_1, \ \varphi_2)(2) = -\xi$
(c) $W(\varphi_1, \ \varphi_2)(2) = \xi e$ (d) $W(\varphi_1, \ \varphi_2)(2) = -\xi e$

5. For each $n \in \mathbb{Z}^+$ the ODE $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ has a solution
(a) $P_n(x) = \dfrac{1}{2^n n!} \dfrac{d^n}{dx^n}(x^2-1)^{n+1}$ (b) $P_n(x) = \dfrac{1}{2^n n!} \dfrac{d^n}{dx^n}(x^2+1)^n$
(c) $P_n(x) = \dfrac{1}{2^n n!} \dfrac{d^n}{dx^n}(x^2-1)^n$ (d) $P_n(x) = \dfrac{1}{2^n n!} \dfrac{d^n}{dx^n}(x-1)^{2n}$

6. Let $P_n(x)$ be $n-$th Legendre polynomial. Then
(a) $P_n(-1) = (-1)^n$ (b) $P_n(-x) = (-1)^{n+1} P_n(x)$ (c) $P_n(-x) = (-1)^{n-1} P_n(x)$ (d) none

7. Let P_n be $n-$th Legendre polynomial. Then for $n = 11$
(a) $\displaystyle\int_{-1}^{1} P_{11}^2(t)dt = \frac{2}{21}$ (b) $\displaystyle\int_{-1}^{1} P_{11}^2(t)dt = \frac{2}{23}$ (c) $\displaystyle\int_{-1}^{1} P_{11}^2(t)dt = \frac{1}{21}$ (d) $\displaystyle\int_{-1}^{1} P_{11}^2(t)dt = \frac{1}{23}$

8. A basis for the solution space of the equation $x^2y'' + (2x^3 - x^2 + x)y' - (5x+1)y = 0, \ x > 0$ for constants c, c_k and C_k is given by
(a) $\varphi_1 = \sum_{k=0}^{\infty} c_k x^{k+1}, \ \varphi_2 = \sum_{k=0}^{\infty} C_k x^{k-1} + c\varphi_1(x)$
(b) $\varphi_1 = \sum_{k=0}^{\infty} c_k x^{k+1}, \ \varphi_2 = \sum_{k=0}^{\infty} C_k x^{k-1} + c \log x$
(c) $\varphi_1 = \sum_{k=0}^{\infty} c_k x^{k+1}, \ \varphi_2 = \sum_{k=0}^{\infty} C_k x^{k-1} + c\varphi_1(x) \log x$
(d) $\varphi_1 = \sum_{k=0}^{\infty} c_k x^{k+1}, \ \varphi_2 = \sum_{k=0}^{\infty} C_k x^{k} + c\varphi_1(x) \log x$

9. Let $u(t)$ be Heaviside step function. Then the inverse Laplace transform $\mathcal{L}^{-1}\left\{\dfrac{e^{-\pi s}}{(s+1)^2+1}\right\}$ is equal to
(a) $u(t+\pi)e^{-(t-\pi)} \sin(t-\pi)$ (b) $u(t-\pi)e^{-(t-\pi)} \sin(t-\pi)$ (c) $u(t-\pi)e^{(t-\pi)} \sin(t-\pi)$ (d) none

10. Choose the correct answer regarding Bessel's function $J_\alpha(x)$

(a) $2\alpha J_\alpha(x) = x(J_{\alpha-1}(x) - J_{\alpha+1}(x)$ (b) $J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^{m-1}}{m!\Gamma(1+m)} \left(\frac{x}{2}\right)^{2m}$

(c) $J_2(x) = \frac{4}{x}J_1(x) - J_0(x)$ (d) $J_3(x) = \left(\frac{8}{x^2} - 1\right) J_1(x) - \frac{4}{x}J_0(x)$

11. The value of $\int_0^\infty (xy-1)e^{-xy}dx,\ y > 0$ is

(a) $\frac{1-y}{y^2}$ (b) 0 (c) 1 (d) $\frac{1}{y^2}$

12. The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ sums to

(a) $\frac{\pi^2}{3}$ (b) $\frac{\pi^2}{4}$ (c) $\frac{\pi^2}{5}$ (d) $\frac{\pi^2}{6}$

13. If $\mathcal{L}\{f(x)\} = F(s)$ then $\mathcal{L}\{f(ax)e^{ax}\},\ s - a > 0$, is

(a) $F(s-a)$ (b) $F\left(\frac{s-a}{a}\right)$ (c) $\frac{1}{a}F\left(\frac{s-a}{a}\right)$ (d) $\frac{1}{a}F(s-a)$

14. Let $u(t) := \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t \le 0 \end{cases}$ and $\mathcal{L}\{f(t)\} = F(s)$. Then inverse Laplace transform $\mathcal{L}^{-1}\{F(s)e^{-as}\}$, $a > 0$ is equal to

(a) $u(t+a)f(t-a)$ (b) $u(t-a)f(t-a)$ (c) $u(t-a)f(t+a)$ (d) $u(t+a)f(t+a)$

15. The dirac delta function $\delta(t-a)$ is defined for every real function f which is continuous in a neighborhood of $t = a$ and every set A containing this neighborhood such that

(a) $\int_A f(t)\delta(t-a) = e^a f(a)$ (b) $\int_A f(t)\delta(t-a) = e^{-a} f(a)$

(c) $\int_A f(t)\delta(t-a) = f(a)$ (d) $\int_A f(t)\delta(t-a) = -f(a)$

16. The surface defined by implicit equation $F(x^2, y^2+z^2) = 0,\ p > 0$ defines the first order PDE given by

(a) $x + pz = 0$ (b) $x + qz = 0$ (c) $y + pz = 0$ (d) $y + qz = 0$

17. General solution of the PDE $2p + 3q - z = 0$ is

(a) $z = ce^x\varphi(3x-2y)$ (b) $z = ce^x\varphi(3x+2y)$ (c) $z = ce^x\varphi(-3x-2y)$ (d) $z = ce^{-x}\varphi(3x-2y)$

18. $z = \varphi(x-ct) + \psi(x+ct),\ c > 0$ is a solution of the PDE

(a) $\frac{\partial z}{\partial x} = \frac{1}{c}\frac{\partial z}{\partial t}$ (b) $\frac{\partial z}{\partial x^2} = \frac{1}{c}\frac{\partial z}{\partial t}$ (c) $\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$ (d) none

19. Let $\frac{a_0}{2} + \sum_1^\infty a_n\cos(nx) + \sum_1^\infty b_n\sin(nx)$ be a fourier series of a function f. Then

(a) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = 0$ (b) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n \neq 0$ (c) $\lim_{n\to\infty} a_n \neq \lim_{n\to\infty} b_n$ (d) none

20. Fourier series of the function $f(x) := \begin{cases} \sin(2x) & \text{if } -\pi \le x \le \pi, \\ 0 & \text{otherwise} \end{cases}$ is

(a) 1 (b) $1 + \sin(x) - 2\sin(2x)$ (c) $\sin(2x)$ (d) does not exist

Key:MTL132

1.d	12.d
2.c	13.c
3.c	14.b
4.c	15.c
5.c	
6.a	16.d
7.b	17.a
8.c	18.c
9.b	
10.d	19.a
11.a	20.c