Name.............................. **Roll No.........................**

Department of Mathematics
Guru Nanak Dev University, Amritsar-143005
Quiz Test in Calculus on Manifolds: M.Sc.Math (Hons.) Semester II

April 18, 2011 **Max. Marks 10**
Course Code#MTL453 **Time Allowed: 60 Minutes**

Note: Read the instructions carefully:
★ Attempt any 20 questions by ticking ✓ *only* one of the four choices (a), (b), (c), and (d) for each question below.
★ Response to any question marked for more than one choice will not be counted for any score.
★ Only the first 20 responses would be counted for the final score.
★ A negative marking for the number of questions attempted exceeding 20 (if any) would be made at a scale of 1/4.

1. If $f : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at a and $g : \mathbb{R}^m \to \mathbb{R}^p$ is differentiable at $f(a)$ then $g \circ f$ is differentiable at
(a) a (b) $f(a)$ (c) $g(f(a))$ (d) none

2. If $p : \mathbb{R}^2 \to \mathbb{R}$ s.t. $p(x, y) = xy$ then
(a) $Dp(a, b) = bx+ay$ (b) $Dp(a, b)(x, y) = bx-ay$ (c) $Dp(a, b)(x, y) = bx+ay$ (d) none of these

3. If $f : \mathbb{R}^{n+m} \to \mathbb{R}^p$ is a bilinear map then
(a) $Df(a, b)(x, y) = f(a, y) - f(x, b)$ (b) $Df(a, b)(x, y) = f(a, y) + f(x, b)$
(c) $Df(a, b)(x, y) = f(a, x) + f(y, b)$ (d) none of these.

4. The function $f(x) := \begin{cases} e^{-1/x} & \text{if } x \in (-1, 1) \\ 0 & \text{otherwise} \end{cases}$ is
(a) real analytic but not C^∞ (b) C^∞ but not real analytic
(c) C^∞ and real analytic (d) none of these

5. If $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) := \begin{cases} x^2 \sin\dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Then at point $x = 0$
(a) f' exists and is continuous (b) f' does not exist
(c) f and f' exist and are continuous (d) f' exists but is not continuous

6. Let $f : [0, \pi/2] \times [0, \pi/2] \to \mathbb{R}^2$ be defined by $f(x) := (\sin x_1, \cos x_2)$ where $x = (x_1, x_2) \in [0, \pi/2] \times [0, \pi/2]$. Then for all $x,\ y$ in the domain of f
(a) $\|f(x) - f(y)\| \leq 2\|x - y\|$ (b) $\|f(x) - f(y)\| \leq 4\|x - y\|$
(c) $\|f(x) - f(y)\| \leq 8\|x - y\|$ (d) none

7. If $f : \mathbb{R} \to \mathbb{R}$ be such that $f'(a) \neq 0$ for all $a \in \mathbb{R}$. Then f is
(a) bijective (b) injective but not surjective (c) surjective but not injective (d) none

8. Let P and P' be two partitions of a rectangle $A \subset \mathbb{R}^n$ and $f : A \to \mathbb{R}$ be a map. Then
(a) $L(f, P') < U(f, P)$ (b) $L(f, P') \geq U(f, P)$
(c) $L(f, P') \leq U(f, P)$ (d) $L(f, P') > U(f, P)$

9. The function $f(x) := \begin{cases} 0 & \text{if } x \in \mathbb{Q} \\ 1 & \text{if } x \notin \mathbb{Q} \end{cases}$ is
(a) integrable (b) differentiable (c) continuous (d) discontinuous

10. A bounded function $f : A \to \mathbb{R}$ is integrable iff it is
(a) discontinuous almost everywhere (b) continuous almost everywhere
(c) continuous (d) discontinuous

11. Choose the correct statement for any $x, y \in \mathbb{R}^n$
(a) $< x, y >= \dfrac{|x + y|^2 + |x - y|^2}{4}$ (b) $< x, y >= \dfrac{|x + y|^2 + |x - y|^2}{2}$
(c) $< x, y >= \dfrac{|x + y|^2 - |x - y|^2}{4}$ (d) none

12. For any $x, y \in \mathbb{R}^n$ one of the following holds
(a) $\Big||x|-|y|\Big| \leq |x|-|y|$ (b) $\Big||x|-|y|\Big| < |x|-|y|$ (c) $\Big||x|-|y|\Big| \leq |x-y|$ (d) $\Big||x|-|y|\Big| \geq |x-y|$

13. A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ is angle preserving if for all $x, y \in \mathbb{R}^n - \{0\}$
(a) $\angle(T(x), T(y)) = \angle(x, y)$ (b) $\angle(T(x), T(y)) = \angle(x, y)$ & T is surjective
(c) $\angle(T(x), T(y)) = \angle(x, y)$ & T is injective (d) none

14. Consider order topology on $\mathbb{R}$. Then
(a) $\text{int}(\mathbb{Q}) = \mathbb{R} = \overline{\mathbb{Q}}$ (b) $\text{int}(\mathbb{Q}) = \mathbb{Q}$ and $\overline{\mathbb{Q}} = \mathbb{R}$ (c) $\text{int}(\mathbb{Q}) = \emptyset$ and $\overline{\mathbb{Q}} = \mathbb{R}$ (d) none

15. In the Euclidean topology on $\mathbb{R}$ and $\mathbb{R}^2$
(a) $\mathbb{R}$ and $\mathbb{R} - \{0\}$ are connected (b) $\mathbb{R}^2$ and $\mathbb{R}^2 - \{(0,0)\}$ are connected
(c) $\mathbb{R}$ and $\mathbb{R} - \{0\}$ are not connected (d) $\mathbb{R}^2$ and $\mathbb{R}^2 - \{(0,0)\}$ are not connected

16. Let $f : X \to Y$ be continuous. Then for every open $V \subset Y$
(a) $f^{-1}(V)$ is closed in X (b) $\exists$ open $U \subset X$ s.t. $f^{-1}(V) \subset U$
(c) $\exists$ open $U \subset X$ s.t. $f(U) \subset V$ (d) none

17. $\det : \Lambda^n(\mathbb{R}^n)$ is defined as an alternating $n-$tensor such that for any n vectors $v_i = \sum_{j=1}^n v_{ij} e_j$ where $\{e_j\}$ is the standard basis of $\mathbb{R}^n$ and
(a) $\det(v_1, \ldots, v_n) := \displaystyle\sum_{\sigma \in S_n} \text{sgn}\sigma \; v_{\sigma(1)1} v_{\sigma(2)2} \cdots v_{\sigma(n)n}$
(b) $\det(v_1, \ldots, v_n) := \displaystyle\sum_{\sigma \in S_{n+1}} \text{sgn}\sigma \; v_{\sigma(1)1} v_{\sigma(2)2} \cdots v_{\sigma(n)n}$
(c) $\det(v_1, \ldots, v_n) := \dfrac{1}{n!} \displaystyle\sum_{\sigma \in S_n} \text{sgn}\sigma \; v_{\sigma(1)1} v_{\sigma(2)2} \cdots v_{\sigma(n)n}$ (d) none

18. A map $f : A \to \mathbb{R}$ is continuous at a point a if
(a) $o(f, a)$ is bounded (b) $o(f, a) = 0$ (c) $o(f, a) < 0$ (d) $o(f, a) > 0$

19. If $f : A \to \mathbb{R}$ is bounded and $B := \{x \in A \mid o(f, x) \geq \epsilon\}$. Then B is
(a) closed and bounded (b) closed but not bounded
(c) bounded but not closed (d) neither closed nor bounded

20. The map $f(x) := \begin{cases} x & x \in \mathbb{Q} \\ 0 & x \notin \mathbb{Q} \end{cases}$ is
(a) not continuous (b) continuous at one point (c) bounded (d) continuous

21. Dimension of the vector space $\Lambda^2(\mathbb{R}^n)$ of the alternating $2-$tensors on $\mathbb{R}^n$ is
(a) $\dfrac{n(n-1)}{3}$ (b) n^2 (c) $\dfrac{n(n-1)}{2}$ (d) none

22. The expression $Alt(Alt(\omega \otimes \eta) \otimes \theta)$ is equal to
(a) $Alt(\omega \otimes \eta \otimes \theta)$ (b) $Alt(\omega \otimes \eta) \otimes \theta$ (c) $\omega \otimes Alt(\eta \otimes \theta)$ (d) none

23. Choose the correct statement
(a) $f^*(\omega \wedge \eta) = f^*\omega \wedge \eta$ (b) $f^*(\omega \wedge \eta) = f^*\omega \wedge f^*\eta$ (c) $\omega \wedge \eta = -\eta \wedge \omega$) (d) none

24. Let $\omega \in \Lambda^n(\mathbb{R}^n)$ be volume element determined by inner product $< \;\; >$ and orientation μ. Let $w_1, dots, w_n \in \mathbb{R}^n$ and define $g_{ij} :=< w_i, w_j >$. Then
(a) $|\omega(w_1, \ldots, w_n)| = \det(g_{ij})$ (b) $|\omega(w_1, \ldots, w_n)| = \det(g_{ij})^2$
(c) $|\omega(w_1, \ldots, w_n)| = n \det(g_{ij})$ (d) $|\omega(w_1, \ldots, w_n)|^2 = \det(g_{ij})$

25. Let $f(x, y, z) := (x^2 + y - z, y - x)$ then $f'(1, 0, 0)$ is
(a) $\begin{pmatrix} -2 & 1 & -1 \\ -1 & 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 1 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

Key:MTL453

1.a	11.c	21.c
2.c	12.c	22.a
3.b	13.c	23.b
4.b	14.c	24.d
5.d	15.b	25.d
6.c	16.c	
7.b	17.a	
8.c	18.b	
9.d	19.a	
10.b	20.b	