Name................................. **Roll No...........................**

Department of Mathematics
Guru Nanak Dev University, Amritsar-143005
Quiz Test in Differential Geometry: M.Sc.Math (Hons.) Semester II

April 15, 2011 **Max. Marks 10**
Code#MTL455 **Time Allowed: 60 Minutes**

Note: Read the instructions carefully:
★ Attempt any 20 questions by ticking ✓ *only* one of the four choices (a), (b), (c), and (d) for each question below.
★ Response to any question marked for more than one choice will not be counted for any score.
★ Only the first 20 responses would be counted for the final score.
★ A negative marking for the number of questions attempted exceeding 20 (if any) would be made at a scale of 1/4.

1. Choose the correct statement
   (a) If $\alpha = O(\beta)$ then $\alpha = o(\beta)$ (b) If $\alpha = o(\beta)$ then $\alpha = O(\beta)$
   (c) $O(\alpha) + O(\alpha) = o(\alpha)$ (d) $o(\alpha) + O(\alpha) = o(\alpha)$

2. The Clover Leaf defined by parametric equations $x = \cos 3t \cos t,\ y = \cos 3t \sin t,\ z = 0,\ 0 \leq t \leq \pi$ represents
   (a) a simple arc (b) a parametric curve (c) simple arc & parametric curve (d) none

3. Two parametric representations $\mathbf{r} = \tau^3\mathbf{a}$ and $\mathbf{r} = t\mathbf{a}$ such that $\mathbf{a}$ is a constant vector and $\tau,\ t \in (-\infty, \infty)$ are
   (a) equivalent (b) having singularities (c) regular (d) none of these.

4. Curvature of the helix $x = 2\cos t,\ y = 2\sin t,\ z = 3t$ at point $t = \pi/6$ is
   (a) $\dfrac{2}{\sqrt{13}}$ (b) $\dfrac{2}{13}$ (c) $\dfrac{3}{\sqrt{13}}$ (d) $-\dfrac{2}{13}$

5. The arc length of the curve $\mathbf{r} := (t - \sin t, 1 - \cos t, 0)$ between $0 \leq t \leq \pi/2$ is
   (a) 4-2$\sqrt{2}$ (b) 4+2$\sqrt{2}$ (c) 4 (d) $2\sqrt{2} - 1$

6. Singular points on the curve $x = t - \sin t,\ y = \sin t,\ z = 0$ for all $n \in \mathbb{Z}$ are
   (a) $t = 2n\pi$ (b) $t = n\pi$ (c) $t = (2n+1)\dfrac{\pi}{2}$ (d) none

7. The osculating plane of the curve $x = t^2,\ y = t - t^2,\ z = 2t$ is given by
   (a) $2X - 2Y + Z = 0$ (b) $2X + 2Y + Z = 0$ (c) $2X + 2Y - Z = 0$ (d) $-2X + 2Y + Z = 0$

8. In Frenet trihedron $[\hat{\mathbf{t}},\ \hat{\mathbf{n}},\ \hat{\mathbf{b}}]$ if $\hat{\mathbf{n}}$ is independent of the natural parameter then the parametric curve is a
   (a) circle (b) straight line (c) helix (d) none of these

9. The natural equation $\kappa = 2\tau$ represents
   (a) circle (b) sphere (c) generalized helix (d) straight line segment

10. The representation $x = \rho\cos\theta,\ y = \rho\sin\theta,\ z = 0$ of the plane $\mathbb{R}^2$ is
    (a) regular (b) singular (c) regular at origin (d) singular at origin

11. The surface of revolution of the cartesian curve $y^2 = x,\ z = 0$ about the $y-$axis is given by
    (a) $z^2 + x^2 = y^4$ (b) $x^2 + y^2 = z^4$ (c) $y^2 + z^2 = x^4$ (d) $z^2 + x^2 = y^2$

12. Envelope of one-parameter family of spheres of constant radius 2 and center on a circle of radius 4 is a
    (a) sphere (b) cone (c) cylinder (d) torus

13. A regular ruled smooth surface $\mathbf{r} := \mathbf{p}(u^1) + u^2\mathbf{I}(u^1)$is developable iff
(a) $\dot{\mathbf{p}} \cdot \dot{\mathbf{I}} \times \mathbf{I} \neq 0$ (b) $\dot{\mathbf{p}} \cdot \dot{\mathbf{I}} \times \mathbf{I} = 0$ (c) $\dot{\mathbf{p}} \times \dot{\mathbf{I}} \times \mathbf{I} \neq \mathbf{0}$ (d) $\dot{\mathbf{p}} \times \dot{\mathbf{I}} \times \mathbf{I} = \mathbf{0}$

14. The first fundamental form of unit sphere in spherical polar coordinates (θ, φ), $\theta \in (-\pi/2,\ \pi/2)$ is given by :
(a) $ds^2 = (d\theta)^2 - \cos^2\theta(d\varphi)^2$ (b) $ds^2 = (d\theta)^2 + \cos^2\varphi(d\varphi)^2$
(c) $ds^2 = (d\theta)^2 + \cos^2\theta(d\varphi)^2$ (d) $ds^2 = (d\theta)^2 - \cos^2\varphi(d\varphi)^2$

15. Orthogonal trajectories of the family of rulings of the cylinder $x = a\cos u^1,\ y = a\sin u^1,\ z = u^2$ lie on the surface
(a) $x =$ constant (b) $y =$ constant (c) $z =$ constant (d) $x + y =$ constant

16. Surface area of a parametric surface $\mathbf{r} = \mathbf{r}(u^1, u^2)$ over a rectangle Ω on it is given by
(a) $\int_\Omega g(u^1, u^2)du^1du^2$ (b) $\int_\Omega \sqrt{g(u^1, u^2)du^1du^2}$ (c) $\int_\Omega \sqrt{g(u^1, u^2)}du^1du^2$ (d) none

17. Gaussian curvature of a smooth surface is given by
(a) $K = \dfrac{g}{b}$ (b) $K = gb$ (c) $K = \dfrac{b}{g}$ (d) $K = \dfrac{b}{\sqrt{g}}$

18. The Christöffél symbols of second kind Γ^k_{ij} of the plane $\mathbb{R}^2$ in cylindrical polar coordinates are
(a) all zero (b) zero except Γ^1_{22} and Γ^2_{12} (c) nonzero except Γ^1_{22} and Γ^2_{12} (d) all nonzero

19. One of the following geometric objects is not an invariant under a change of curvilinear coordinates:
(a) g_{ij} (b) $\int_\Omega g du^1du^2$ (c) $\dfrac{b}{g}$ (d) δ^j_i

20. Second fundamental form of the surface of revolution $x = 2u\cos v,\ y = 2u\sin v,\ z = 2v$ is
(a) $\text{II} = 4(du)^2 + (4u^2+1)(dv)^2$ (b) $\text{II} = 4(du)^2 + (4u^2-1)(dv)^2$
(c) $\text{II} = 8(du)^2 + (8u^2+1)(dv)^2$ (d) $\text{II} = 8(dv)^2$

21. Radius of the osculating sphere of the curve $x = 2\cos t,\ y = 2\sin t,\ z = 2t$ at point $t = \pi/2$ is
(a) $\dfrac{1}{2}$ (b) 4 (c) $\dfrac{1}{4}$ (d) 2

22. The envelope of one parameter family of unparallel planes $\mathbf{N}(u) \cdot \mathbf{R} = \delta(u)$, $\mathbf{N}(u)\perp$ to the planes for $\mathbf{N} \cdot \dot{\mathbf{N}} \times \ddot{\mathbf{N}} = 0$, is
(a) a cylinder (b) a cone (c) a twisted curve (d) all of these

23. The implicit equation $F(x, y, z) = 0$ always represents
(a) a simple sheet of s surface (b) a surface (c) a curve (d) none of these

24. Let $f : M \to M'$ be a mapping of differentiable manifolds. Then f is an embedding if
(a) f is an immersion (b) f is a local homeomorphism
(c) f is a bijective immersion (d) none of these

25. Differential equation for a regular smooth parametric curve without rectification points is given by
(a) $\begin{pmatrix} \hat{\mathbf{t}}' \\ \hat{\mathbf{n}}' \\ \hat{\mathbf{b}}' \end{pmatrix} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & -\tau \\ 0 & \tau & 0 \end{pmatrix}\begin{pmatrix} \hat{\mathbf{t}} \\ \hat{\mathbf{n}} \\ \hat{\mathbf{b}} \end{pmatrix}$ (b) $\begin{pmatrix} \hat{\mathbf{t}}' \\ \hat{\mathbf{n}}' \\ \hat{\mathbf{b}}' \end{pmatrix} = \begin{pmatrix} 0 & -\kappa & 0 \\ \kappa & 0 & -\tau \\ 0 & \tau & 0 \end{pmatrix}\begin{pmatrix} \hat{\mathbf{t}} \\ \hat{\mathbf{n}} \\ \hat{\mathbf{b}} \end{pmatrix}$
(c) $\begin{pmatrix} \hat{\mathbf{t}}' \\ \hat{\mathbf{n}}' \\ \hat{\mathbf{b}}' \end{pmatrix} = \begin{pmatrix} 0 & -\kappa & 0 \\ \kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}\begin{pmatrix} \hat{\mathbf{t}} \\ \hat{\mathbf{n}} \\ \hat{\mathbf{b}} \end{pmatrix}$ (d) $\begin{pmatrix} \hat{\mathbf{t}}' \\ \hat{\mathbf{n}}' \\ \hat{\mathbf{b}}' \end{pmatrix} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}\begin{pmatrix} \hat{\mathbf{t}} \\ \hat{\mathbf{n}} \\ \hat{\mathbf{b}} \end{pmatrix}$

Key:MTL455

1.b	11.b	21.b
2.b	12.d	22.a
3.a	13.b	23.d
4.b	14.c	24.c
5.a	15.c	25.d
6.a	16.c	
7.c	17.c	
8.b	18.b	
9.c	19.a	
10.d	20.a	