Department of Mathematics
Guru Nanak Dev University, Amritsar-143005
Major Test-2011
B.Sc.(Hons.)Physics, Semester-II

Subject: Mathematics II | **Max. Marks 50**
Code: MTL132 | **Time allowed: 03 Hours**

Note: All questions carry equal marks.

1. Derive the Laplace equation $\nabla^2\varphi = 0$ in orthogonal spherical polar coordinates (r, θ, φ) of a point in $\mathbb{R}^3$. (5)

2. Obtain a general solution of the Laplace equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0,\ u(x,0) = f(x)$, using fourier transforms where $-\infty < x < \infty,\ 0 < y < \infty$. (5)

3. Prove that $\displaystyle\int_{-\infty}^{\infty} e^{-x^2} = \sqrt{\pi}$. Using it, find fourier transform of the function $f(x) := e^{-4x^2}$ defined on $-\infty < x < \infty$. (5)

4. If $\mathcal{F}[f](\omega)$ is the Fourier transform of a function $f(x)$, then prove that:

   (i) $\mathcal{F}[f](-\omega) = \overline{\mathcal{F}[f](\omega)}$ and (ii) $\mathcal{F}[f(ax+b)](\omega) = \dfrac{1}{|a|} e^{\frac{i\omega b}{a}} \mathcal{F}[f(x)]\left(\dfrac{\omega}{a}\right),\ a \neq 0$ (5)

5. Calculate the improper integral
$$\int_0^{\infty} \frac{dt}{(a^2+t^2)(b^2+t^2)},\ a, b > 0,$$
using the generalized Plancherel identity. (5)

6. Using fourier transforms, solve the integral equation: $\displaystyle\int_{-\infty}^{\infty} \frac{af(t)}{(x-t)^2 + a^2} dt = \frac{b}{x^2+b^2}$, for the function $f(x)$ where $0 < a < b$. (5)

7. Show that a fourier series representation of the function $f(x) = x$ on $[-\pi,\ \pi]$ is given by
$$f(x) \sim \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx.$$
Using it prove that $\dfrac{\pi}{2\sqrt{2}} = 1 + \dfrac{1}{3} - \dfrac{1}{5} - \dfrac{1}{7} + \dfrac{1}{9} + \dfrac{1}{11} - \dfrac{1}{13} - \dfrac{1}{15} + \cdots$. (5)

8. Determine fourier series expansion for the function $f(x) := \begin{cases} \sin 2x, & 0 \le x \le \frac{\pi}{2}, \\ 0, & \frac{\pi}{2} \le x \le \pi. \end{cases}$. Using it evaluate the sum $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(4n^2-1)^2}$. (5)

9. Prove the identity $1 + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \dfrac{1}{4^4} + \cdots \infty = \dfrac{\pi^4}{90}$ using a fourier series expansion for function $f(x) := x^2$ where $-\pi \le x \le \pi$. (5)

10. Obtain a fourier series solution of the diffusion equation
$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{1}{k}\frac{\partial \varphi}{\partial t},\ k > 0,\ 0 < t < \infty,\ -L < x < L,$$
using variable separable method s.t. $u(-L,t) = u(L,t)$ and $\dfrac{\partial u}{\partial x}(-L,t) = \dfrac{\partial u}{\partial x}(L,t)$. (5)