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Department of Mathematics
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Quiz Test in Group Theory: M.Sc. (Hons.) Mathematics, Semester I

October 20, 2011 — Max. Marks 10
MTL402 — Max. time allowed: 60 Minutes

Note: Read the instructions carefully:

- Attempt any 20 questions by ticking ✓ *only* one of the four options (a), (b), (c), and (d) for each question below.
- Response to any question marked for more than one option will not be counted for any score.
- Only the first 20 responses would be counted for the final score.
- A negative marking for the number of questions attempted exceeding 20 (if any) would be made at a scale of $1/4$.

1. One of the following is a composition series for the cyclic group $\mathbb{Z}/10\mathbb{Z}(\cong \mathbb{Z}_{10})$:
   (a) $1 \trianglelefteq \langle\bar{4}\rangle \trianglelefteq \langle\bar{2}\rangle \trianglelefteq \mathbb{Z}/10\mathbb{Z}$ (b) $1 \trianglelefteq \langle\bar{2}\rangle \trianglelefteq \mathbb{Z}/10\mathbb{Z}$
   (c) $1 \trianglelefteq \langle\bar{3}\rangle \trianglelefteq \mathbb{Z}/10\mathbb{Z}$ (d) $1 \trianglelefteq \langle\bar{6}\rangle \trianglelefteq \mathbb{Z}/10\mathbb{Z}$

2. Let G be a group and $x \in G$ s.t. $|x| = n$, then $|x^t|$ for all $t \in \mathbb{Z}$ is
   (a) $\dfrac{2n}{\gcd(n,t)}$ (b) $\dfrac{t}{\gcd(n,t)}$ (c) $\dfrac{n}{\gcd(n,|t|)}$ (d) None of these

3. Total number of group homomorphisms from $\mathbb{Z}_{36} \to \mathbb{Z}_{81}$
   (a) 81 (b) 9 (c) 36 (d) none of these.

4. Order of the multiplicative group $(\mathbb{Z}/n\mathbb{Z})^\times$ is
   (a) $2\varphi(n)$ (b) $\varphi(n)/2$ (c) $n\varphi(n)/2$ (d) $\varphi(n)$

5. Let $\sigma \in S_9$ be a permutation s.t. $\sigma := (123456789)$. Then $|\sigma^6|$ is
   (a) 1 (b) 3 (c) 6 (d) 9.

6. For each divisor $|\bar{d}|$ of $\gcd(m,n)$, $m,n \in \mathbb{Z}^+, m>1, n>1$, $\bar{d} \in Z_n$, the total number of group homomorphisms $\varphi : \mathbb{Z}_m \to \mathbb{Z}_n$, sending $\bar{1} \mapsto \bar{d}$ is
   (a) 1 (b) m (c) n (d) $\gcd(m,n)$.

7. The group $\mathrm{Aut}(Q_8)$ of automorphisms of the Quaternion group Q_8 is
   (a) Q_8 (b) D_8 (c) S_3 (d) S_4

8. One of the following groups has trivial center
   (a) $\mathbb{R}$ (b) S_4 (c) $\mathbb{Z}_8$ (d) $\mathbb{Z} \times \mathbb{Z}$

9. If $n \geq m$, then the number of $m-$cycles in the permutation group S_n is given by:
   (a) $\dfrac{n(n-1)\cdots(n-m+1)}{m}$ (b) $\dfrac{n(n-1)\cdots(n-m+1)}{2}$
   (c) $\dfrac{n(n-1)\cdots(n-m+1)}{n}$ (d) none of these

10. Let $|G| < \infty$, $H \leq G$; under the group action $\circ : G \times G/H \to G/H$ defined by $g \circ (xH) := (gx)H$, the orbit of H is
    (a) H (b) G (c) G/H (d) none of these

11. Let $X \neq \emptyset$ be set and $\circ : G \times X \to X$ be a group action. Then for all $x \in X$
    (a) $|G| = |X|$ (b) $|G| = |\mathcal{O}_x|/|\mathrm{Stab}(x)|$ (c) $|G| = |\mathcal{O}_x||\mathrm{Stab}(x)|$ (d) none of these

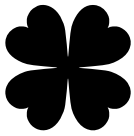
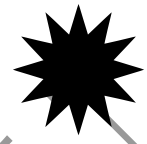
12. Up to isomorphism, the total number of distinct groups of order 4 is equal to
    (a) 1 (b) 2 (c) 3 (d) 4

13. If a group G is non-abelian then
    (a) $\mathrm{Aut}(G) \cong \{1\}$ (b) $\mathrm{Inn}(G) \cong \{1\}$ (c) $\mathrm{Inn}(G) \ncong \{1\}$ (d) nothing can be said

14. A group isomorphism between two groups always preserves:
    (a) the group operation (b) the order of the elements
    (c) both the assertions (a) & (b) (d) none of these

15. The total number of elements of order 2 in S_4 is
(a) 1 (b) 3 (c) 9 (d) none of these

16. The symmetric group S_n is abelian for
(a) $n = 2$ (b) $n = 3$ (c) $n = 4$ (d) for all $n \geq 3$

17. The group of symmetries of 'rotations' and 'out of plane flip' in one of the following geometrical plane figures is the dihedral group D_8
(a)  (b)  (c)  (d) 

18. The Klein-4 group does not satisfy one of the following:
(a) It is abelian (b) It has 3 elements of order 2
(c) It has no element of order 4 (d) It is cyclic

19. A group is finite abelian if and only if
(a) it is cyclic (b) it is simple
(c) it has a composition series (d) none of these

20. If A and B are subgroups of G such that $A \subset N_G(B)$ then
(a) $AB \neq BA$ (b) $AB \trianglelefteq B$ (c) $AB \trianglelefteq A$ (d) $B \trianglelefteq AB$

21. The order of the element $x = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ of the group $GL_2(\mathbb{C})$ is
(a) 1 (b) 2 (c) 4 (d) 8

22. The group $\mathrm{Aut}(\mathbb{R})$ is isomorphic to
(a) $\mathbb{R}$ (b) $\{1\}$ (c) $\mathbb{R}/2\pi\mathbb{Z}$ (d) none of these

23. Let G be a finite non abelian group. If two elements from G are chosen at random, the maximum probability that they commute is
(a) 1 (b) 1/8 (c) 3/8 (d) 5/8

24. Let $|G| = 2k$ for some positive integer k. Then
(a) G is always abelian (b) G has an element of order 2
(c) G is always non-abelian (d) G can't have any normal subgroup

25. Let G be a finite cyclic group. Then the number of its generators is equal to
(a) $|G|$ (b) $\varphi(|G|)$ (c) $\varphi(|G|)/2$ (d) none of these

26. Under the action of a finite group G onto itself via conjugation, the number of distinct singleton orbits is
(a) $|C_G(x)|$ for some $x \in G$ (b) $|Z(G)|$ (c) $\sum_{x \notin Z(G)} |G|/|C_G(x)|$ (d) none of these

27. The torsion group $\mathrm{Tor}(\mathbb{R}/\mathbb{Z})$ is equal to
(a) $\mathbb{R}/\mathbb{Z}$ (b) $\mathbb{Q}/\mathbb{Z}$ (c) $\mathbb{Z}/n\mathbb{Z}$ (d) none of these

28. Minimum number of generators of the symmetric group S_n is
(a) 1 (b) 2 (c) 3 (d) Infinite

29. Minimum number of generators of the additive group $\mathbb{Q}$ of all rational numbers is
(a) 1 (b) 2 (c) 3 (d) Infinite

30. Up to isomorphism there are m distinct groups of order 15 where m is equal to
(a) 1 (b) 2 (c) 3 (d) none of these

1.*b*	11.*c*	21.*b*
2.*c*	12.*b*	22.*b*
3.*b*	13.*c*	23.*d*
4.*d*	14.*c*	24.*b*
5.*b*	15.*c*	25.*b*
6.*a*	16.*a*	26.*b*
7.*d*	17.*c*	27.*b*
8.*b*	18.*d*	28.*b*
9.*a*	19.*c*	29.*d*
10.*c*	20.*d*	30.*a*