Department of Mathematics
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Quiz Test in Abstract Algebra: M.Sc.(Hons.) I

October 2010 | **Max. Marks 10**
Code#MLT402 | **Time Allowed: 60 Minutes**

Note: Read the instructions carefully:

★ Attempt any 20 questions by ticking ✓ *only* one of the four options (a), (b), (c), and (d) for each question below.

★ Response to any question marked for more than one option will not be counted for any score.

★ Only the first 20 responses would be counted for the final score.

★ A negative marking for the number of questions attempted exceeding 20 (if any) would be made at a scale of 1/4.

1. One of the following is a composition series for the dihedral group D_8 :
   (a) $1 \trianglelefteq \langle s \rangle \trianglelefteq \langle s, r^2 \rangle \trianglelefteq D_8$ (b) $1 \trianglelefteq \langle s \rangle \trianglelefteq \langle r^2 \rangle \trianglelefteq D_8$
   (c) $1 \trianglelefteq \langle s \rangle \trianglelefteq \langle r \rangle \trianglelefteq D_8$ (d) $1 \trianglelefteq \langle r \rangle \trianglelefteq \langle s, r^3 \rangle \trianglelefteq D_8$

2. Let k and n be two positive integers and $n > 2$. Then quotient group $D_{2n}/\langle r^k \rangle \cong D_{2k}$ if
   (a) $\gcd(n,k) = 1$ (b) $1 < \gcd(n,k) < k$. (c) $\gcd(n,k) = k$ (d) None of these

3. Total number of homomorphisms from the additive group $(\mathbb{Z}, +)$ to the additive group of residue classes modulo 81 is
   (a) 81 (b) 54 (c) 2 (d) none of these.

4. Order of the multiplicative group $(\mathbb{Z}/1000\mathbb{Z})^{\times}$ is
   (a) 200 (b) 300 (c) 400 (d) 500

5. Total number of homomorphisms from the cyclic group $\mathbb{Z}_{21} \rightarrow$ to the cyclic group $\mathbb{Z}_{70}$ is
   (a) 3 (b) 7 (c) 21 (d) 70.

6. Total number of homomorphisms from the cyclic group $\mathbb{Z}_{70} \rightarrow$ to the cyclic group $\mathbb{Z}_{21}$ is
   (a) 3 (b) 7 (c) 21 (d) 70.

7. The dihedral group D_{2048} is
   (a) Simple (b) Not simple (c) Abelian and simple (d) Non-abelian and simple

8. If $\mathbb{F}_p$ is a finite field for some prime p. Then the order of the group $GL_2(\mathbb{F}_p)$ is
   (a) $p^4 - p^3 - p^2 - p$ (b) $p^4 - p^3 - p^2 + p$ (c) $p^4 - p^3 + p^2 - p$ (d) $p^4 + p^3 - p^2 - p$

9. If $n \geq m$, then the number of $m-$cycles in the permutation group S_n is given by:
   (a) $\dfrac{n(n-1)\cdots(n-m+1)}{m}$ (b) $\dfrac{n(n-1)\cdots(n-m+1)}{2}$
   (c) $\dfrac{n(n-1)\cdots(n-m+1)}{n}$ (d) none of these

10. If $n \geq 4$, then the number of permutations of S_n which are product of two disjoint cycles is:
(a) $\frac{n(n-1)(n-2)(n-3)}{n}$ (b) $\frac{n(n-1)(n-2)(n-3)}{2}$
(c) $\frac{n(n-1)(n-2)(n-3)}{4}$ (d) $\frac{n(n-1)(n-2)(n-3)}{8}$

11. The number of distinct normal subgroups in an infinite cyclic group upto isomorphism is
(a) one (b) countably infinite (c) uncountable (d) none of these

12. Let G be a group. Then the quotient group $G/Z(G)$ is isomorphic to
(a) $\mathrm{Aut}(G)$ (b) $\mathrm{Inn}(G)$ (c) $\mathrm{Aut}(G)/\mathrm{Inn}(G)$ (d) none of these

13. If a group G is non-abelian then
(a) $\mathrm{Aut}(G) \cong \{1\}$ (b) $\mathrm{Inn}(G) \cong \{1\}$ (c) $\mathrm{Inn}(G) \not\cong \{1\}$ (d) nothing can be said

14. A group homomorphism between two groups always preserves:
(a) the group operation (b) the order of the elements
(c) both the assertions (a) & (b) (d) none of these

15. The total number of injective maps from a finite set with n elements onto itself is
(a) $n!$ (b) n (c) $n!/2$ (d) $n/2$

16. The symmetric group S_n is abelian for
(a) $n = 2$ (b) $n = 3$ (c) $n = 4$ (d) for all $n \geq 3$

17. The product of cycles $(123)(4) \circ (234)(1) \circ (12)(34) \circ (1234)$ in the symmetric group S_4 is
(a) $(12)(34)$ (b) $(123)(4)$ (c) (1234) (d) none of these

18. The Klein-4 group does not satisfy one of the following:
(a) It is abelian (b) It has 3 elements of order 2
(c) It has no element of order 4 (d) It is cyclic

19. Jordan Holder's theorem is not always true for
(a) the additive group $\mathbb{Z}/n\mathbb{Z}$ for all $n \in \mathbb{Z}^+$ (b) finite abelian groups
(c) finite non-abelian groups (d) infinite groups

20. If A and B are subgroups of G such that $A \subset N_G(B)$ then
(a) $AB \neq BA$ (b) $AB \trianglelefteq B$ (c) $AB \trianglelefteq A$ (d) $B \trianglelefteq AB$

21. The order of the element $x = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ of the group $GL_2(\mathbb{R})$ is
(a) 1 (b) 2 (c) 3 (d) 4

22. The subgroup $\{z \in \mathbb{C} \mid |z| = 1\}$ of the multiplicative group $\mathbb{C}^\times$ is isomorphic to
(a) $\mathbb{C}$ (b) $\{-1, 1\}$ (c) $\mathbb{R}/\mathbb{Z}$ (d) none of these

23. If p is a prime divisor of the order of a finite group G then
(a) G has an element of order p^2 (b) G is always abelian
(c) G has an element of order p (d) no conclusion can be drawn

24. Let G be a group. Then tick the correct statement
(a) G is always abelian (b) G has a normal subgroup (c) G is always non-abelian (d) G does not have any normal subgroup

25. Let G be a finite cyclic group and $x \in G$ be its generator. Then
(a) $|x| < |G|$ (b) $|x| \geq |G|$ (c) $|x| = |G|$ (d) $|x| > |G|$

26. Under the action of a finite group G onto itself via conjugation, the number of distinct singleton orbits is
(a) $|C_G(x)|$ for some $x \in G$ (b) $|Z(G)|$ (c) $\sum_{x \notin Z(G)} |G|/|C_G(x)|$ (d) none of these

27. Choose the correct statement from the following about a non-abelian group G:
(a) $Z(G) = G$ (b) $Z(G) \subseteq G$ (c) $Z(G) \subsetneq G$ (d) none of these

28. Number of distinct generators of an infinite cyclic group is
(a) 1 (b) 2 (c) 3 (d) Infinite

29. The symmetric group S_3 has a composition series of length
(a) 1 (b) 2 (c) 3 (d) 4

30. If $x,\ y$ are any two elements of an abelian group G with finite orders s.t. $|x| = m$ and $|y| = n$ then
(a) $|xy| = mn$ (b) $|xy| = \gcd(m,n)$ (c) $|xy| \leq mn/\gcd(m,n)$ (d) $|xy| = |G|/(mn)$

1.a	11.a	21.d
2.c	12.b	22.c
3.a	13.c	23.c
4.c	14.a	24.b
5.b	15.a	25.c
6.b	16.a	26.b
7.b	17.c	27.c
8.b	18.d	28.b
9.a	19.d	29.c
10.d	20.d	30.c