## Department of Mathematics Guru Nanak Dev University, Amritsar-143005 Quiz Test in Abstract Algebra: M.Sc.(Hons.) I

 $\begin{array}{c} \text{October 2010} \\ \text{Code} \# \text{MLT402} \end{array}$ 

Max. Marks 10 Time Allowed: 60 Minutes

*Note:* Read the instructions carefully:

- $\bigstar$  Attempt any 20 questions by ticking  $\checkmark$  only one of the four options (a), (b), (c), and (d) for each question below.
- \* Response to any question marked for more than one option will not be counted for any score.
- $\bigstar$  Only the first 20 responses would be counted for the final score.
- ★ A negative marking for the number of questions attempted exceeding 20 (if any) would be made at a scale of 1/4.
  - 1. One of the following is a composition series for the dihedral group  $D_8$ :
    - (a)  $1 \le \langle s > \le \langle s, r^2 \rangle \le D_8$  (b)  $1 \le \langle s > \le \rangle < r^2 > \le D_8$
    - (c)  $1 \le \langle s \rangle \le \langle r \rangle \le D_8$
- 2. Let k and n be two positive integers and n > 2. Then quotient group  $D_{2n}/\langle r^k \rangle \cong D_{2k}$  if
  - (a)  $\gcd(n,k) = 1$  (b)  $1 < \gcd(n,k) < k$ . (c)  $\gcd(n,k) = k$  (d) None of these
- 3. Total number of homomorphisms from the additive group  $(\mathbb{Z},+)$  to the additive group of residue classes modulo 81 is
  - (a) 81 (b) 54 (c) 2 (d) none of these.
- 4. Order of the multiplicative group  $(\mathbb{Z}/1000\mathbb{Z})^{\times}$  is
  - (a) 200 (b) 300 (c) 400 (d) 500

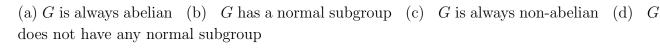
- 5. Total number of homomorphisms from the cyclic group  $\mathbb{Z}_{21} \to \text{to the cyclic group } \mathbb{Z}_{70}$  is
  - (a) 3 (b) 7 (c) 21 (d) 70.
- 6. Total number of homomorphisms from the cyclic group  $\mathbb{Z}_{70} \to \text{to the cyclic group } \mathbb{Z}_{21}$  is (a) 3 (b) 7 (c) 21 (d) 70.
- 7. The dihedral group  $D_{2048}$  is
  - (a) Simple (b) Not simple (c) Abelian and simple (d) Non-abelian and simple
- 8. If  $\mathbb{F}_p$  is a finite field for some prime p. Then the order of the group  $GL_2(\mathbb{F}_p)$  is (a)  $p^4 - p^3 - p^2 - p$  (b)  $p^4 - p^3 - p^2 + p$  (c)  $p^4 - p^3 + p^2 - p$  (d)  $p^4 + p^3 - p^2 - p$
- 9. If  $n \ge m$ , then the number of m-cycles in the permutation group  $S_n$  is given by:

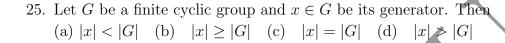
  (a)  $\frac{n(n-1)\cdots(n-m+1)}{m}$  (b)  $\frac{n(n-1)\cdots(n-m+1)}{2}$ 
  - $\frac{n(n-1)\cdots(n-m+1)}{n}$  (d) none of these

10.	If $n \ge 4$ , then the number of permutations of $S_n$ which are product of two disjoint cycles is (a) $\frac{n(n-1)(n-2)(n-3)}{n}$ (b) $\frac{n(n-1)(n-2)(n-3)}{2}$ (c) $\frac{n(n-1)(n-2)(n-3)}{4}$ (d) $\frac{n(n-1)(n-2)(n-3)}{8}$
11.	The number of distinct normal subgroups in an infinite cyclic group upto isomorphism is (a) one (b) countably infinite (c) uncountable (d) none of these
12.	Let $G$ be a group. Then the quotient group $G/Z(G)$ is isomorphic to (a) $\operatorname{Aut}(G)$ (b) $\operatorname{Inn}(G)$ (c) $\operatorname{Aut}(G)/\operatorname{Inn}(G)$ (d) none of these
13.	If a group $G$ is non-abelian then (a) $\operatorname{Aut}(G)\cong\{1\}$ (b) $\operatorname{Inn}(G)\cong\{1\}$ (c) $\operatorname{Inn}(G)\ncong\{1\}$ (d) nothing can be said
14.	A group homomorphism between two groups always preserves:  (a) the group operation  (b) the order of the elements  (c) both the assertions (a) & (b) (d) none of these
15.	The total number of injective maps from a finite set with $n$ elements onto itself is (a) $n!$ (b) $n$ (c) $n!/2$ (d) $n/2$
16.	The symmetric group $S_n$ is abelian for (a) $n=2$ (b) $n=3$ (c) $n=4$ (d) for all $n \geq 3$
17.	The product of cycles $(123)(4) \circ (234)(1) \circ (12)(34) \circ (1234)$ in the symmetric group $S_4$ is (a) $(12)(34)$ (b) $(123)(4)$ (c) $(1234)$ (d) none of these
18.	The Klein-4 group does not satisfy one of the following:  (a) It is abelian  (b) It has 3 elements of order 2  (c) It has no element of order 4  (d) It is cyclic
19.	Jordan Holder's theorem is not always true for (a) the additive group $\mathbb{Z}/n\mathbb{Z}$ for all $n \in \mathbb{Z}^+$ (b) finite abelian groups (c) finite non-abelian groups (d) infinite groups
20.	If $A$ and $B$ are subgroups of $G$ such that $A \subset N_G(B)$ then (a) $AB \neq BA$ (b) $AB \trianglelefteq B$ (c) $AB \trianglelefteq A$ (d) $B \trianglelefteq AB$
21.	The order of the element $x=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ of the group $GL_2(\mathbb{R})$ is (a) 1 (b) 2 (c) 3 (d) 4

22. The subgroup  $\{z \in \mathbb{C} \mid |z| = 1\}$  of the multiplicative group  $\mathbb{C}^{\times}$  is isomorphic to (a)  $\mathbb{C}$  (b)  $\{-1,1\}$  (c)  $\mathbb{R}/\mathbb{Z}$  (d) none of these

<ul> <li>23. If p is a prime divisor of the order of a finite group G then</li> <li>(a) G has an element of order p² (b) G is always abelian</li> <li>(c) G has an element of order p (d) no conclusion can be drawn</li> </ul>
24. Let $G$ be a group. Then tick the correct statement







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(a) |C_G(x)| for some x \in G (b) |Z(G)| (c) \sum_{x \notin Z(G)} |G|/|C_G(x)| (d) none of these
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27. Choose the correct statement from the following about a non-abelian group G:

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(a) Z(G) = G (b) Z(G) \subseteq G (c) Z(G) \subsetneq G (d) none of these
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- 28. Number of distinct generators of an infinite cyclic group is
  (a) 1 (b) 2 (c) 3 (d) Infinite
- 29. The symmetric group  $S_3$  has a composition series of length (a) 1 (b) 2 (c) 3 (d) 4
- 30. If x, y are any two elements of an abelian group G with finite orders s.t. |x| = m and |y| = n then

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(a) |xy| = mn (b) |xy| = \gcd(m, n) (c) |xy| \le mn/\gcd(m, n) (d) |xy| = |G|/(mn)
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1.a	11.a	21.d
2.c	12.b	22.c
3.a	13.c	23.c
4.c	14.a	24.b
5.b	15.a	25.c
6.b	16.a	26.b
7.b	17.c	27.c
8.b	18.d	28.b
9.a	19.d	29.c
10.d	20.d	30.c

