Roll No.——————

Major Test

M.Sc. Mathematics(Hons.) Semester-I

2014

Algebra I — Max. Marks 50

MTL402 — Max. time allowed: 03 hours

Note:

(1) All questions carry equal marks.

(2) The symbol G will denote an abstract group.

1. (a) Prove that a group of order 255 is cyclic. (1)

   (b) State the Dickson's theorem. (1)

   (c) What is the number of elements of order 2 in S_5? (1)

   (d) Let $H \leq S_9$ such that $(12)(345) \in H$. Determine $|H \cap A_9|$. (1)

   (e) How many elements of order 9 are there in the group $Z_6 \times Z_3 \times Z_2$? (1)

2. (a) Prove that a subgroup of a cyclic group is cyclic. (3)

   (b) Prove that $G/Z(G) \cong \mathrm{Inn}G$. (2)

3. (a) Prove that $Z(S_n)$ is trivial for all $n \geq 3$. (3)

   (b) Identify the quotient group $D_8/Z(D_8)$. (2)

4. (a) Let $|G| = 2n$ where $n > 1$ is an odd integer. Prove that G is solvable. (3)

   (b) Prove that conjugate elements in S_n have same cycle type. (2)

5. (a) Prove that $G' \trianglelefteq G$ and that G/G' is abelian. (3)

   (b) If $H \mathrm{char} K$ and $K \trianglelefteq G$, prove that $H \trianglelefteq G$. (2)

6. (a) Prove that every finite abelian group is isomorphic to a direct product of its Sylow subgroups. (3)

   (b) Classify upto isomorphism, all abelian groups of order 630. (2)

7. Let G be a finite group and p is a prime divisor of $|G|$. Prove that G has ta least one element of order p. (5)

8. Let G be a finite group. Prove that the number n_p of Sylow-p-subgroups of G is equal to $[G : N_G(H)]$ and that $n_p \equiv 1 \mod p$ for each prime divisor of $|G|$. (5)

9. Classify upto isomorphism, all groups of order 66. (5)

10. Assuming that A_5 is simple, prove that A_n is simple for all $n \geq 6$. (5)