Roll No. ——————

Topology

M.Sc.(Hons.)Mathematics, Semester-IV

Major Test

May 14, 2013 | Max. Marks 50

MTL551 | Time allowed: 03 hour

Instructions to the students:

i)Read each question carefully before attempting the solution.

ii)All questions carry equal marks.

1. Let X be a first countable topological space and A be a subset of X. Prove that a sequence of points of A converges to a point $x \in X$ if and only if $x \in \bar{A}$. (5)

2. (a) Let $f : [0,1] \to [0,1]$ be continuous. Prove that there is a point $x \in [0,1]$ such that $f(x) = x$. (2)

   (b) Prove that the general linear group $GL_n(\mathbb{R})$ as a subspace of $\mathbb{R}^{n^2}$ is not connected. (3)

3. (a) Prove that the product of two path connected topological spaces is path connected. (2)

   (b) Prove that a subspace of a completely regular space is completely regular. (3)

4. (a) Prove that a locally compact Hausdörff space is completely regular. (2)

   (b) Let X be a topological space such that for every collection $\mathcal{C}$ of closed sets in X having the finite intersection property, $\cap_{C \in \mathcal{C}} \neq \emptyset$. Prove that X is compact. (3)

5. Let $\mathcal{A}$ be an open covering of a metric space (X, d). If X is compact, prove that there is a real number $\delta > 0$ such that each subset A of X with diameter less than δ, is contained in some member of $\mathcal{A}$. (5)

6. Prove that every compact Hausdörff space is normal. (5)

7. Let for any closed subspace A of a topological space X, and any continuous function $f : A \to \mathbb{R}$, there is a continuous extension $g : X \to \mathbb{R}$ such that $g(x) = f(x)$ for all $x \in A$. Prove that X is normal. (5)

8. Let X be a normal space. Prove that there is a continuous map $f : X \to [0,1]$ such that for any two disjoint closed subsets A and B of X, $f(A) = \{0\}$ and $f(B) = \{1\}$. (5)

9. Prove that a regular space with countable basis is always metrizable. (5)

10. Let X be a set with the following properties (i) For any collection $\mathcal{A}$ of subsets of X having finite intersection property(FIP), there is a collection $\mathcal{D} \supset \mathcal{A}$ of subsets of X such that $\mathcal{D}$ is maximal with respect to FIP. (ii) Any finite intersection of members of $\mathcal{D}$ is in $\mathcal{D}$ (iii) If A is a subset of X that intersects every element of $\mathcal{D}$ then $A \in \mathcal{D}$. Assuming (i), (ii), and (iii), *prove that an arbitrary product of compact topological spaces is compact.* (5)