Score ☐ **Name:** _____ **Roll No.** _____

Quiz–Analysis I, October 2015

Max. Marks 10 Max. time allowed: 20 Minutes

Note: Read the instructions carefully:

✽Attempt all questions by writing one of the four choices A, B, C, and D for each question in the box ☐ provided. Use only ball-point pen or gel-ink pen for the purpose.

✽Response to any question marked outside the box ☐ will not be counted for any score.

1. Let $f : (\mathbb{R}, d) \to \mathbb{R}$ be such that $d(x, y) = \begin{cases} 0, & if\ x = y; \\ 1, & if\ x \neq y. \end{cases}$ Choose the incorrect statement

   A. f is bounded B. $(\mathbb{R}, d)$ is bounded C. $(\mathbb{R}, d)$ is compact D. $\{0\}$ is open in $(\mathbb{R}, d)$ ☐

2. Define $S = \{f \mid f : \mathbb{N} \to \{1, 2\}\}$. Then the set S is

   A. countable B. uncountable C. finite D. none of these ☐

3. Let $D : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$, such that $D(x_1 \times y_1, x_2 \times y_2) = |x_1 - x_2| + |y_1 - y_2|$. Let d be the Euclidean metric on $\mathbb{R}^2$. Then for any $\epsilon > 0$ and $x, y \in \mathbb{R}^2$,

   A. $\sqrt{2}d(x, y) \leq D(x, y)$ B. $D(x, y) \leq d(x, y)$

   C. $B_d(x, \epsilon) \subseteq B_D\left(x, \frac{\epsilon}{\sqrt{2}}\right)$ D. $B_D(x, \epsilon) \subseteq B_d(x, \epsilon)$ ☐

4. Let $S = \{p \in \mathbb{Q} \mid 2 < p^2 < 3\}$, where $\mathbb{Q}$ is a subspace of $\mathbb{R}$. Then the set

   A. S is bounded but not closed in $\mathbb{Q}$ B. S is compact in $\mathbb{Q}$

   C. S is connected in $\mathbb{Q}$ D. S is closed and bounded in $\mathbb{Q}$ ☐

5. Let $f : \mathbb{X} \to X$, such that $f(f(x)) = x$. Then

   A. f is injective but not surjective B. f is surjective but not injective

   C. f is injective as wee as surjective D. none of these ☐

6. One of the following statements is not true for the Cantor's ternary set $C \subseteq \mathbb{R}$:

   A. C is perfect B. C is compact C. C is closed D. C is countable ☐

7. Let $A = \left\{(x, y) \in \mathbb{R}^2 \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ a \neq b\right\}$, $B = \left\{(x, y) \in \mathbb{R}^2 \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1,\ a \neq b\right\}$,

   $C = \left\{(x, y) \in \mathbb{R}^2 \mid ax + by + 5 = 0\right\}$, $D = \left\{(x, y) \in \mathbb{R}^2 \mid x = y^2\right\}$,

   $E = \left\{(x, y) \in \mathbb{R}^2 \mid x^3 + y^3 = 1\right\}$, then

   A. C and D are compact but $A, B,$ and E are not compact

   B. A and E are compact but $B, C,$ and D are not compact

   C. A and B are compact but $C, D,$ and E are not compact

   D. $A,$ B and E are compact but C, and D are not compact ☐

8. Let $P = \left\{1 + \frac{1}{n} \mid n \in \mathbb{N}\right\}$, $Q = \left\{\frac{1}{n} + \frac{1}{m} \mid n, m \in \mathbb{N}\right\}$, and $R = [0, 1] \cap \mathbb{Q}$ in $\mathbb{R}$. Then

   A. $P^\circ = Q^\circ = R^\circ = \emptyset$ B. $\bar{P} = P \cup \{0\}$, $\bar{Q} = P \cup \{0\}$, and $\bar{R} = [0, 1]$

   C. $P' = \{0\}$, $Q' = P \cup \{0\}$, and $R' = C$ D. none of these ☐

9. Choose the correct statement

   A. $\mathbb{Q}$ has least upper bound property B. $\mathbb{Z}$ has least upper bound property

   C. $\mathbb{R}$ has the well ordereing property D. $\mathbb{R}$ is always compact ☐

10. A set S is infinite if and only if

    A. $|S|$ is equal to a proper subset of S B. S is uncountable

    C. $|S| = |\mathbb{N}|$ D. none of these ☐

Checked by _____